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# Calculus

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**Seventh Edition**

**Elliott Mendelson, PhD**

*Professor of Mathematics  
Queens College*

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